• PHYSICS •

The Physics of Crumpled Paper

Hayley Chu, *China*

Crumpling is the process whereby a sheet of paper undergoes deformation to yield a three-dimensional structure comprising a random network of ridges and facets with variable density. A regular sheet of paper can be easily torn and is very flimsy, yet when a sheet of paper is crumpled into a ball, the crumpled paper becomes much sturdier and has a large compressive strength, which is a material's maximum compressive load divided by its cross-sectional area (Compressive strength, n.d.). A crumpled paper ball is estimated to contain around 75-90% air, and cannot easily be compacted further (Croll, Twohig, & Elder, 2019; Keim, 2011).

This phenomenon has been studied extensively by scientists due to its peculiar structure, using methods ranging from "kvetching"—"kvetch" is a Yiddish word that means "complain" and literally translates to "squeeze" or "press" (Roberts, 2018), and refers to crushing the material, crumpled paper, in a cylindrical container—to X-ray microtomography, an imaging technique that assembles three-dimensional images from thousands of two-dimensional, cross-section snapshots (Keim, 2011). However, the physics behind paper crumpling is not yet completely understood by scientists even today.

Crumpling as a type of deformation is present not just in the crumpling of a piece of paper and throwing it in a bin, but in many areas in nature, such as the initial unfolding of an insect's wing, and even how DNA packs into a cell nucleus (Roberts, 2018). Given that there are various implications of crumpling dynamics, this essay aims to explore why crumpled paper has such a complicated and strong structure, and the real-life applications of this.

Research question

What are the possible reasons behind the peculiar structure of crumpled paper, and what are the implications of this?

Analysis

The four-building block structures

Crumpled sheets are made of four main structures, which include the bend, the fold, the developable cone (d-cone) and the stretching ridge. Bending occurs when the two ends of a sheet of paper are placed on top of another so there is a bend in the middle. When force is applied to the bend, the paper will be folded, creating a fold. When stretching is concentrated at one point and the rest of the sheet is bending in a cone-like shape, the vertex of the sheet is called a d-cone (Cambou, On the crumpling of thin sheets, 2014). When two d-cones are created in a sheet, stretching will occur between the d-cones and form a stretching ridge. These four structures are shown in Figure 1 below.

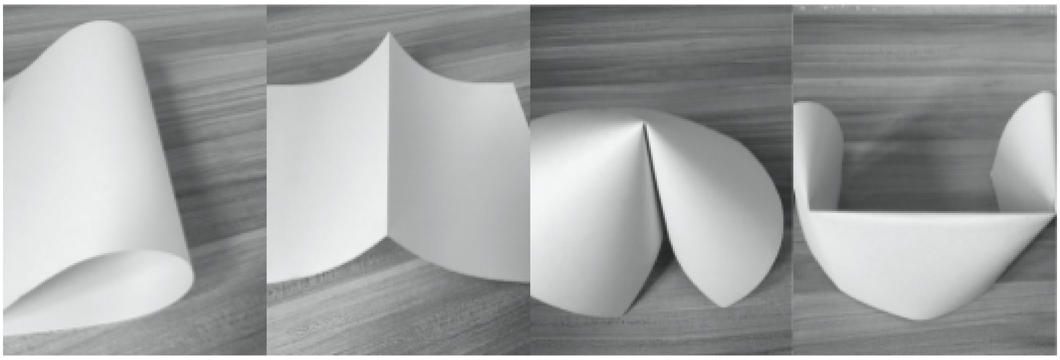

Figure 1: (From left to right) The bend, the fold, the d-cone and the stretching ridge.

Croll, Twohig and Elder conducted experiments on these four structures to find the force response of crumpled materials and to determine the strongest structures and why they were the strongest. These experiments used thin films created with a range of thicknesses (from 100 nm to 1 mm) from two different but well-characterised polymeric materials, polycarbonate (PC), a glassy polymer (a polymer is a substance with a molecular structure made of a large number of similar units bonded together) with a Young's modulus of 1.6 GPa and polydimethylsiloxane (PDMS), an elastomer (an elastomer is a polymer that has elastic properties) with Young's modulus of 1.69 MPa.

The films were crumpled by hand and placed between two parallel glass plates, which comprise the compression cell. Laser scanning confocal microscopy of the film allowed direct observation of the internal crumple structure throughout the experiment.

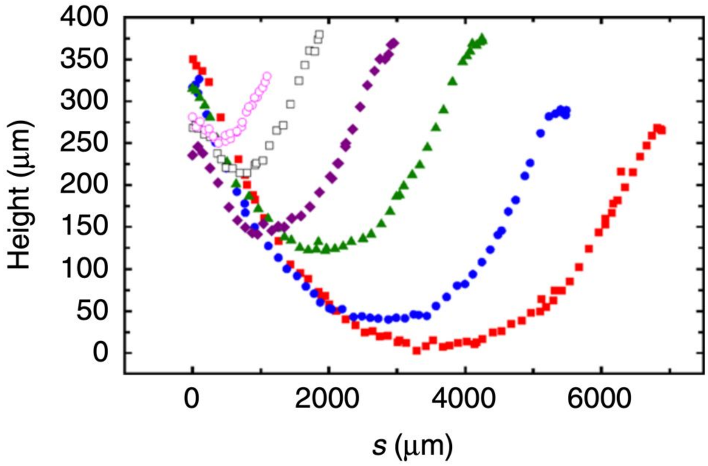

Figure 2: Ridge height of the PDMS sample as a function of s, a coordinate running along the peak curvature of the ridge (Croll, Twohig, & Elder, 2019).

Croll et al. concluded that energy stored in a ridge is thought to be stored mostly along the ridge peak. This is because in Figure 2, the curves, similar to a parabola, have a dip in the middle, which means that the length of the ridge is longer than needed to directly connect the two d-cones.

They also discovered that in the PC stretched ridges, the d-cones are fixed so while compression attempts to push them together, the d-cones cannot move so only bending occurs in the structure before the ridge buckles. This shows that the ridges deliver energy into the available soft modes (bending), which progressively stiffen until the ridge structure collapses (Croll, Twohig, & Elder, 2019). The stretching ridges have a high buckling strength, which is a quantity that measures how much force a material can withstand before buckling. Therefore, bending in the stretching ridge is responsible for the strength of both elastic and plastic crumples.

Comparison to paper folding and X-ray microtomography experiments

Cambou and Menon conducted X-ray microtomography experiments in order to study the three-dimensional structure of crumpled sheets. Since X-rays would directly penetrate paper and would not yield adequate results, they crumpled aluminium sheets into balls by hand and used a computerised tomography (CT) scanner to scan images of the interior of the crumpled balls. They found that the sheet creates parallel stacks, or folds, which is surprising given that crumpling a sheet, especially by hand appears to be a very random process. They also discovered that a crumpled ball is most dense in its outer region, and least dense in the core (Keim B. , 2011), which would explain why there is a greater probability of 6-stacks, or stacks of 6 folded layers, forming in the outer layers of the sphere than 3-stacks, or stacks of 3 folded layers (Cambou & Menon, Three-dimensional structure of a sheet crumpled into a ball, 2011).

Previous studies like the one conducted by Croll et al. have shown that ridges help contribute to crumpled paper's strength due to their high buckling strength, and this is further amplified by this X-ray microtomography experiment since the ridges in the crumpled ball create tension in the crease and make it extremely hard to compress. However, Cambou and Menon did make an unexpected discovery.

There is an old myth that a sheet of A4 paper cannot be folded in half more than 7 times (the current world record for the most paper folds is 12) because with each fold, the thickness of the paper doubles, so as one makes more folds there will be impossible amounts of energy required to complete the crease. According to Menon, "If you have five things stacked, you've increased the strength 125 times," which means that the force required to fold the paper is proportional to the cube of the number of sheets. Crumpled paper follows a similar power law. The paper ball is harder to deform when crumpled due to the fact that the layers act like folds. Moreover, the layers trap air (as seen from the fact that crumpled paper balls are mostly made up of air), which could be one more variable that adds to the structure's strength (Scrunch time: The peculiar physics of crumpled paper, 2011). As these multiple folded layers add up, they act as structural pillars. Since they are aligned in many different, random directions and are isotropically arranged (Cambou & Menon, Three-dimensional structure of a sheet crumpled into a ball, 2011), they strengthen the crumpled ball in every direction. Thus, no matter which angle you press down on the crumpled ball, you're pressing down against these columns, which resist being crushed in all different directions. Therefore, stacks are another factor contributing to a crumpled paper ball's mechanical rigidity.

Investigation

Effective density

INTRODUCTION

Bansal, Chowdhry and Gyaneshwaran (2018) studied the relationship between the side length of square sheets of paper that were crumpled into paper balls and the diameter of the crumpled balls, thereby calculating the effective density of the crumpled paper balls. This first investigation attempts to imitate their experiment.

HYPOTHESIS

We begin by equating the mass of a paper sheet to the mass of the crumpled paper balls.

$$\rho_p l^2 t = \rho_e \frac{4\pi r^3}{3} \text{ or } \rho_p l^2 t = \rho_e \frac{\pi d^3}{6}$$

Where ρ_p is the measured density of the paper sheet, l is the side length of the square sheet of paper, t is the thickness of the paper, ρ_e is the effective density of the paper ball and d is the diameter of the paper ball.

Rearranging, we have: $d^3 = \frac{\rho_p 6 t l^2}{\rho_e \pi}$

Since $d^3 \propto l^2$, the hypothesis is that the diameter cubed of the paper ball is directly proportional to the side length squared of the paper sheet.

VARIABLES

Independent: The side length squared of the paper sheets.
Dependent: The diameter cubed of the crumpled paper balls.
Controlled: The compression force applied by hand and the material of the paper.

METHODOLOGY

1. Identical sheets of A4 paper were cut into square pieces with different side lengths.
2. These were crumpled by hand to form the shape of a ball.
3. The Vernier calliper and screw gauge were used to measure the diameter of the paper balls.

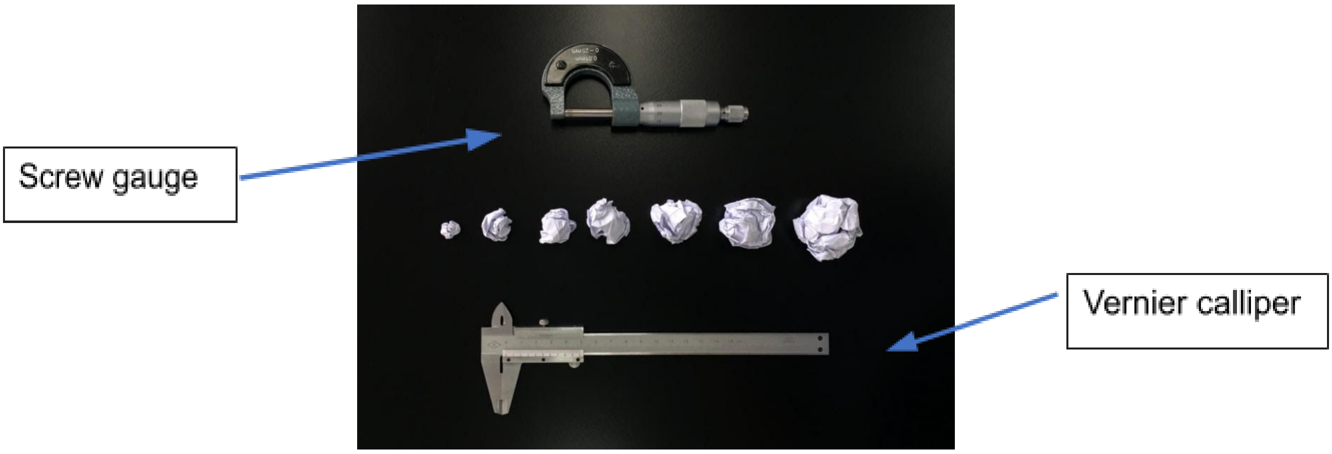

Figure 3: The different sized crumpled paper balls and apparatus set-up.

RESULTS AND DISCUSSION

Side length (cm) $\Delta L = \pm 0.05$ cm	Side length^2 (cm^2) $\Delta L^2 = \pm 0.1$ cm^2	Diameter (cm) $\Delta D = \pm 0.005$ cm	Diameter^3 (cm^3) $\Delta D^3 = \pm 0.015$ cm^3	Effective density (g/ cm^3)
5	25	1.15	1.52	0.2262
8	64	1.80	5.83	0.1510
10	100	2.15	9.94	0.1384
12	144	2.50	15.63	0.1268
15	225	2.90	24.39	0.1269
17	289	3.10	29.79	0.1335
20	400	3.60	46.66	0.1180

Table 1: Raw quantitative data for the effective density experiment.

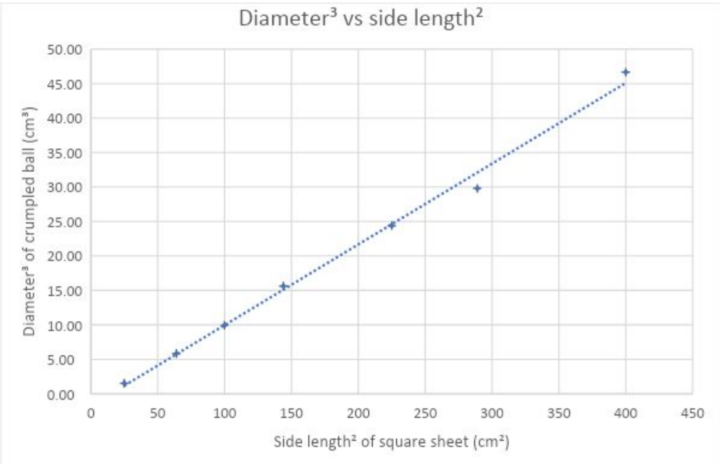

Figure 4: The graph of diameter^3 against side length^2.

The standard dimensions of A4 paper are 21 cm x 29.7 cm x 0.01 cm, and the standard mass is 4.50 g. Thus, the measured density for an A4 sheet of paper is $\rho_p = \frac{mass}{volume} = \frac{4.5}{21 \times 29.7 \times 0.01} = 0.72\ g/cm^3$.
The average effective density was then calculated to be :

$$\rho_e = \frac{6\rho_p l^2 t}{\pi d^3} = 0.15 \pm 0.003\ g/cm^3.$$

It can be seen that the effective density of the crumpled paper balls is 4.8 times less than the measured density of the paper sheet. This supports Croll et al.'s idea that crumpled paper is mostly made up of air. While the diameter cubed of the paper ball varies with the side length squared of the paper sheet, shown in figure 4, the effective density remains roughly constant throughout the experiment and only depends on the compression force applied by hand.

Rearranging the equation again and using the slope of the trendline, the thickness can be calculated by :

$$t = \frac{\rho_p \pi d^3}{6\rho_e l^2} = \frac{\rho_p}{\rho_e} \times \frac{\Delta d^3}{\Delta l^2} \times \frac{\pi}{6} = 4.8 \times 0.1171 \times \frac{\pi}{6}$$
$$= 0.0198 \pm 0.003\ cm$$

This is quite close to the standard thickness of A4 paper, 0.01 cm, thus proving the hypothesis correct. One reason the calculated value for thickness is a little larger than the standard thickness could be due to the fact that the 0.2262 g/cm^3 value for the effective density when the side length is 5 cm is an outlier, and since the diameter cubed is inversely proportional to the effective density, the first value for the diameter cubed could have been smaller, making the trendline slope larger.

This outlier is perhaps due to the fact that when measuring the diameter of the paper ball with the Vernier calliper there could have been accidental force applied to the paper ball, making the diameter measurement smaller than in reality. Another reason for this could be that the crumpling process was conducted entirely by hand, meaning that it was very hard to keep the compacting force constant for all the paper balls. Yet another explanation could be that it was much easier to crumple a smaller piece of paper than a large one, as a larger one would trap more air in its crumpled form. This meant the crumpled paper ball made from the 5 cm paper sheet was actually denser than the ball made from the 20 cm paper sheet. This could be a potential further area for study, to see how the effective density of paper balls varies with the size of the sheet of paper used to crumple it, with very small increments and small sizes.

If this experiment were repeated, there could be more attention paid to whether the Vernier calliper is applying any unwanted force to the paper ball during diameter measurements. Moreover, more side lengths could be tested to ensure that the relationship holds for large sized crumpled paper balls.

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